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Electric Monopoles in Topological Field Theories

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A tensor product generalization of $B \wedge F$ theories is proposed that has a Bogomol'nyi structure. Nonsingular, stable, finite-energy particle-like solutions to the Bogomol'nyi equations are studied. Unlike Yang–Mills(–Higgs) theory, the Bogomol'nyi structure does not appear as a perfect square in the Lagrangian. Consequently, the Bogomol'nyi energy can be obtained in more than one way. The added flexibility permits electric monopole solutions.

1. INTRODUCTION

A Bogomol'nyi structure in a Lagrangian field theory frequently yields classical, nonsingular, stable, finite-energy particle-like solutions to the variational field equations. Moreover, particle-like solutions—called Bogomol'nyi solitons—appear to have far fewer quantum corrections than might usually be expected. For instance, the classical mass spectrum for Bogomol'nyi solitons has no quantum correction; this is due to a general relationship between supersymmetry and the Bogomol'nyi structure (Witten and Olive, 1978; Hlousek and Spector, 1993). Also, for some Bogomol'nyi solitons (e.g., the BPS magnetic monopole) the quantum corrections to the scattering differential cross section have been found to be remarkably and unexpectedly small (Temple-Raston and Alexander, 1993).

In this paper we study a generalization of the $B \wedge F$ topological field theories introduced by Horowitz (1989). These theories are comprised of generally covariant topological gauge field theories. The generalization investigated here uses a tensor-product structure in the Lagrangian to produce a Bogomol'nyi structure in this class of topological field theories. Solitons analogous to the BPS magnetic monopole in Yang–Mills–Higgs theory are

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found. Unlike Yang–Mills–Higgs theory, however, our Lagrangian does not rely on a metric structure. The metric structure used to define the Hodge star-operator in Yang–Mills–Higgs theory is responsible for turning the Bogomol'nyi soliton into a magnetic monopole. Without the metric in the tensor-product theory we shall construct explicit electric monopole solutions to the field equations.

2. TFTs AND BOGOMOL'NYI STRUCTURES

The Lagrangian field theory that forms the basis of our work is given by

$$\mathcal{L}(A, B) = \int_{\mathbf{R}^4} \langle (H^A \otimes I_E) \wedge (I_E \otimes K^B) \rangle - \tfrac{1}{2} \langle (I_E \otimes K^B)^2 \rangle + (A \leftrightarrow B, H^A \leftrightarrow K^B) \tag{1}$$

where H^A and K^B are gauge field curvatures over $\mathbf{R}^4$ taking values in the adjoint bundle E over $\mathbf{R}^4$. Here I_E is the identity transformation on the adjoint bundle E. The tensor products are taken on the Lie algebras, and the wedge products on the forms. The form of this Lagrangian is based on the topological gauge field theories studied some time ago by Horowitz (1989) and the theories of Baulieu and Singer (1988). By introducing two gauge potentials instead of having just one, the four-dimensional gauge field theory examined here contains source-free electrodynamics and Yang–Mills theory (Temple-Raston, 1995).

Our interest in this paper is restricted to topological solitons with rest mass. Therefore without loss of generality we can restrict our investigation to stationary topological solitons. This leads us to dimensionally reduce the four-dimensional theory using a gauge symmetry in time (Forgács and Manton, 1980). We do this now. We denote $\mathbf{R}^4$ quotiented by the time-symmetry by M_3. Let the gauge group equal $U(n)$. Let P be a principal $U(n)$-bundle over the three-manifold M_3. Denote the space of connections on P by $\mathcal{A}(P)$. A vector bundle $E \to M_3$ is associated to the principal bundle P by the adjoint representation. For each connection $A \in \mathcal{A}(P)$ there is an exterior covariant derivative D^A acting on sections of E. The covariant derivative defines a curvature H^A for the vector bundle E by $D^A D^A s = H^A s$, where s is a section of the vector bundle $\pi\colon E \to M_3$. The curvature H^A can be interpreted as a 2-form on M_3 taking values in E. An equivariant Lie algebra-valued Higgs field Φ_A on M_3 is paired with the connection A. In dimensional reduction the Higgs field arises naturally as the component of the vector potential in the direction of the gauge symmetry (Forgács and Manton, 1980). After the reduction of the Lagrangian (1), our starting point now becomes the energy functional $2\pi\mathcal{E}(A, B, \Phi_A, \Phi_B)$, given by

$$\int_{M_3} \langle (I_E \otimes K^B) \wedge (I_E \otimes D^B\Phi_B)\rangle$$
$$- \int_{M_3} \langle (I_E \otimes K^B) \wedge (D^A\Phi_A \otimes I_E)\rangle$$
$$- \int_{M_3} \langle (H^A \otimes I_E) \wedge (I_E \otimes D^B\Phi_B)\rangle + (A \leftrightarrow B, H^A \leftrightarrow K^B, \Phi_A \leftrightarrow \phi_B) \quad (2)$$

In the expression (2) there are two curvatures H^A, K^B and two Higgs fields Φ_A, Φ_B corresponding to two connections $A, B \in \mathcal{A}(P)$. We assume that there is an invariant positive-definite inner product $\langle\cdot\rangle$ on E_x which varies continuously with $x \in M_3$. The last term in (2) symmetrizes the energy functional in the dependent fields. The energy functional is complicated, but as we shall see it inherits useful geometric structure from the four-dimensional theory, presented and studied in Temple-Raston (1995, 1997). In coordinate notation the energy functional $2\pi\mathscr{E}(A, B, \Phi_A, \Phi_B)$ can be rewritten as

$$\int_{M_3} K^a_{[ij}(D^B_{k]}\Phi_B)^b \operatorname{tr}(T^aT^b)$$
$$- \int_{M_3} K^a_{[ij}(D^A_{k]}\Phi_A)^b \operatorname{tr}(T^aI_E) \operatorname{tr}(T^bI_E)$$
$$- \int_{M_3} H^a_{[ij}(D^B_{k]}\Phi_B)^b \operatorname{tr}(T^aI_E) \operatorname{tr}(T^bI_E) + (A \leftrightarrow B, \Phi_A \leftrightarrow \Phi_B) \quad (3)$$

Square brackets denote skew-symmetrization, and Latin subscripts run from 1 to 3. Since the gauge group is $U(n)$ we have used the Killing–Cartan form for the bundle inner product $\langle\cdot\rangle$ normalized so that $\langle I_E^2\rangle = 1$.

By completing the square, we can rewrite the energy functional (2) as

$$2\pi\mathscr{E} = \int_{M_3} \langle (H^A \otimes I_E - I_E \otimes K^B) \wedge (D^A\Phi_A \otimes I_E - I_E \otimes D^B\Phi_B)\rangle$$
$$- \int_{M_3} \langle (D^A\Phi_A \otimes I_E) \wedge (H^A \otimes I_E)\rangle + (A \leftrightarrow B, \Phi_A \leftrightarrow \Phi_B) \quad (4)$$

Let E_A and E_B denote the vector bundle E equipped with either the connection A or B, respectively. We recall that the curvature of the tensor product bundle $E_A \otimes E_B^*$ is given by (Kobayashi, 1987)

$$\Omega_{E_A \otimes E_B^*} = H^A \otimes I_E - I_E \otimes K^B$$

By defining $\Phi \equiv \Phi_A \otimes I_E - I_E \otimes \Phi_B$, then

$$D_{E_A \otimes E_B^*}\Phi = D^A\Phi_A \otimes I_E - I_E \otimes D^B\Phi_B$$

The first integral in (4) is thus a topological invariant: the winding number. The Bogomol'nyi equations determined by the energy functional (4) are

$$\begin{aligned} H^A \otimes I_E &= I_E \otimes K^B \\ D^A\Phi_A \otimes I_E &= I_E \otimes D^B\Phi_B \end{aligned} \tag{5}$$

The first equation in (5) is clearly a zero-curvature condition on the tensor product bundle $E_A \otimes E_B^*$. Reintroducing a coordinate system, we can rewrite the equations in (5) as

$$\begin{aligned} H_{ij} &= K_{ij} = F_{ij}(iI) \\ D_i^A\Phi_A &= D_i^B\Phi_B = E_i(iI) \end{aligned} \tag{6}$$

F and E are a real-valued two-form and one-form on M_3, respectively, and I is the identity matrix. Written in this form, solutions to the first equation in (6) are recognized—geometers call them projectively flat connections (Kobayashi, 1987). For bundles of rank greater than one, projective flatness is a strong condition (Kobayashi, 1987).

We emphasize that unlike the theory of BPS magnetic monopoles, the first integral in the energy functional (4) is not in the form of a perfect square, and with one or the other Bogomol'nyi equation satisfied the energy functional attains the Bogomol'nyi energy:

$$2\pi\mathscr{E} = \int_{M_3} (D_{[k}^A\Phi_A)^a H_{ij]}^b \operatorname{tr}(T^aT^b) + \int_{M_3} (D_{[k}^B\Phi_B)^a K_{ij]}^b \operatorname{tr}(T^aT^b) \tag{7}$$

The extra flexibility in the topological field theory will lead us to the construction of electric monopole solutions.

3. PROJECTIVELY FLAT SOLITONS

Presumably topological monopoles, if they exist, are analogous to the BPS magnetic monopole field configurations in Yang–Mills–Higgs theory. Therefore we shall use the same symmetry-breaking mechanism (Goddard and Olive, 1978). The solitonic core region is placed at the origin. Let G and G_o be compact and connected gauge groups, where the group G_o is assumed to be embedded in G. It is sufficient to have the Higgs covariantly constant, $D^A\Phi_A = 0$, for the gauge group of the core region G to be spontaneously broken to G_o outside of the core region. In regions far from the core where we assume that $D^A\Phi_A = 0$, it can be shown that

$$H^A = \Phi_A F_A \tag{8}$$

where $F_A \in \Lambda^2(M_3, E_{G_o})$, a two-form on M_3 taking values in the G_o-Lie algebra bundle, denoted by E_{G_o} here (Goddard and Olive, 1978). An equivalent expression to (8) can be written when $D^B\Phi_B = 0$. We shall assume that both Φ_A and $\Phi_B \rightarrow aI_E$ for some constant a, when $r >> 1$ and where spontaneous symmetry breaking has occurred. When $G = U(n)$ and $G_o = U(1)$, F_A becomes a pure imaginary two-form on M_3.

The Bogomol'nyi solitons defined by (6) have an energy (7) topologically fixed by

$$\begin{aligned} 2\pi\mathscr{E} &= \int_{M_3} d(\mathrm{tr}(\Phi_A H^A)) + \int_{M_3} d(\mathrm{tr}(\Phi_B K^B)) \\ &= \int_{S^2} \mathrm{tr}(\Phi_A H^A) + \int_{S^2} \mathrm{tr}(\Phi_B K^B) \end{aligned} \tag{9}$$

where S^2 is a large sphere surrounding the monopole core. Substituting equation (8) into equation (9) and using the asymptotic normalization condition $\langle\Phi^2\rangle = a^2$ for both Higgs fields, we find that the energy is fixed by

$$\frac{a^2}{2\pi}\left(\int_{S^2} F_A + \int_{S^2} F_B\right) \tag{10}$$

We can view F_A and F_B as curvatures on the line bundles $L_A \rightarrow S^2$ and $L_B \rightarrow S^2$ determined by Φ_A and Φ_B, because from equation (8) F_A and F_B are the projections of H^A and K^B on L_A and L_B.

We shall now argue that it is appropriate and natural to interpret $a \int F_A/2\pi$ and $a \int F_B/2\pi$ in (10) as the magnetic charge g and the electric charge q, respectively. This interpretation is arrived at by returning to the four-dimensional action (1) and observing that standard (source-free) electrodynamics is regained when the vector potential B is chosen so that

$$K^B = \pm {*H^A} \tag{11}$$

where a space-time metric is introduced through the Hodge star operator. The variational field equations from the action functional (1),

$$D^A K^B + D^B H^A = 0 \tag{12}$$

become the Yang–Mills equations (for each independent vector potential). A Bogomol'nyi structure similar to that presented above is also present in the four-dimensional theory. The corresponding Bogomol'nyi equations again give projective-flatness,

$$H^A = K^B = iFI$$

where F is a real valued two-form on $\mathbf{R}^4$ (Temple-Raston, 1997). If we take F in

$$F_A = \langle H^A \Phi_A \rangle = -iF\,\mathrm{tr}(\Phi_A)$$

to be the Faraday tensor, then using (11), F_B becomes

$$F_B = \langle {*H^A} \Phi_B \rangle = -i{*F}\,\mathrm{tr}(\Phi_B)$$

With the space-time metric and the Faraday tensor introduced in this way and $\Phi_A = \Phi_B = aI_E$ on S^2, it then follows that $a \int F_A/2\pi = a \int \mathbf{B}\cdot d\mathbf{S}/2\pi$ is the magnetic charge g and $a \int F_B/2\pi = a \int \mathbf{E}\cdot d\mathbf{S}/2\pi$ is the electric charge q. Moreover, the magnetic and electric charges are now seen to be proportional to topological invariants—the Chern numbers associated to complex line bundles with curvatures F_A and F_B, respectively. The Bogomol'nyi energy is given by

$$\mathscr{E} = a^2(c_1(L_A) + c_1(L_B)) = a(g + q) \tag{13}$$

As a result both the solitonic electric and magnetic charges in this theory are 'quantized' at the classical level, and the stability of the topological monopole is assured by (13) if either the electric or magnetic charge is nonvanishing.

Let us now consider nonsingular, particle-like $U(n)$ solutions to *both* Bogomol'nyi equations in (6) that are spontaneously broken in the far-field. From the projective flatness of the curvatures in the Bogomol'nyi equations (6), $H^A = K^B = F(iI)$, and from equation (8) we conclude that $F = \varphi_A F_A = \varphi_B F_B$, where $\Phi_A = \varphi_A(iI)$, $\Phi_B = \varphi_B(iI)$ and φ_A, φ_B are real-valued functions on M_3. The normalization of the Higgs fields implies that $\varphi_A^2 = \varphi_B^2 = a^2$. From this we find that

$$\int_{S^2} F_A = \pm \int_{S^2} F_B \tag{14}$$

Therefore nonsingular, stable, particle-like solutions to both Bogomol'nyi equations (6) are dyons.

To obtain electric monopoles there would appear to be two possibilities, both resulting from a weakening of one or the other Bogomol'nyi equation (6). We do not favor relaxing the projective flatness of the solitons, however, because we then lose mathematical control over the topological properties of the configuration space (Kobayashi, 1987). Instead, we shall maintain projective flatness and let go of $D^A\Phi_A = D^B\Phi_B = E(iI)$, at least asymptotically. Furthermore, since little empirical evidence exists to suggest the independent existence of two gauge potentials, it is desirable to restrict to a smaller set of topological solitons defined by $A = B$. We shall call these solutions 'diagonal projectively-flat solitons.'

4. DIAGONAL PROJECTIVELY FLAT ELECTRIC MONOPOLES

In this section we demonstrate the existence of diagonal projectively-flat $U(2)$ electric monopoles ($A = B$, Φ_A, Φ_B) on $\mathbf{R}^3$ situated at the origin. Following the example of the BPS magnetic monopole, we define the outside of the monopole to be where the gauge field is broken with a covariantly constant Higgs field (Goddard and Olive, 1978). The electric monopoles we will construct have the following properties:

1. $A = B$ are projectively flat over all of $\mathbf{R}^3$ and take values in the Lie algebra of $U(2)$.
2. $A = B$ are asymptotically flat on $\mathbf{R}^3$.
3. Φ_A, Φ_B are sufficiently differentiable functions on $\mathbf{R}^3$ taking values in the Lie algebras of $SU(n)$ and $U(n)$, respectively.
4. $D^B\Phi_B = 0$ and $D^A\Phi_A \neq 0$, asymptotically. When $D^B\Phi_B = 0$, we assume that $\Phi_B = aI_E$ for a nonzero constant a.
5. The electric charge of the monopole is nonzero, and the magnetic charge vanishes.

Condition 4 implies that the gauge symmetry for K^B is broken to $U(1)$ asymptotically, and that the gauge group for H^A is not permitted to break far from the origin. For condition 5, assume that a two-sphere of radius r, S_r^2, lies completely outside the monopole that is centered at the origin. The Bogomol'nyi energy (9) is then given by

$$\mathscr{E} = -\frac{1}{2\pi}\int_{S_r^2} F_B \operatorname{tr}(\Phi_A\Phi_B) - \frac{a^2}{2\pi}\int_{S_r^2} F_B \tag{15}$$

The first integral in (15) is the magnetic charge. The magnetic charge vanishes since Φ_A taking values in the Lie algebra of $SU(2)$ is traceless and $\Phi_B = aI_E$, conditions 3 and 4, respectively. The second term is the topological electric charge; the electric charge must obviously be nonzero for an electric monopole. We shall show that a solution satisfying conditions 1–5 exists.

In general, a $U(2)$ diagonal topological soliton ($A = B$, Φ_A, Φ_B) must be of the form

$$A_j = B_j = \begin{pmatrix} ia_j & -c_j^* \\ c_j & ib_j \end{pmatrix}$$

$$\Phi_A = \begin{pmatrix} i\alpha_A & -\gamma_A^* \\ \gamma_A & i\beta_A \end{pmatrix}, \qquad \Phi_B = \begin{pmatrix} i\alpha_B & -\gamma_B^* \\ \gamma_B & i\beta_B \end{pmatrix} \tag{16}$$

$a_j, b_j, \alpha_A, \beta_A, \alpha_B, \beta_B$ are all real-valued functions on $\mathbf{R}^3$. The first Bogomol'nyi equation in (6) states that the vector potential is projectively flat, $H^A = K^B = F(iI)$. A straightforward calculation of H^A informs us that

$$\begin{aligned} F_{ij} &= \partial_i a_j - \partial_j a_i + i(c_j^* c_i - c_i^* c_j) \\ &= \partial_i b_j - \partial_j b_i - i(c_j^* c_i - c_i^* c_j) \end{aligned} \tag{17}$$

and that

$$\partial_i c_j - \partial_j c_i - i[(c_i a_j - c_j a_i) - (c_i b_j - c_j b_i)] = 0 \tag{18}$$

Equation (17) in coordinate-free notation becomes

$$F = d\mathbf{a} + i\mathbf{c}^* \wedge \mathbf{c} = d\mathbf{b} - i\mathbf{c}^* \wedge \mathbf{c} \tag{19}$$

and implies that $\mathbf{c}^* \wedge \mathbf{c} = 2id(\mathbf{b} - \mathbf{a})$, that is, $\mathbf{c}^* \wedge \mathbf{c}$ is exact. Similarly, equation (18) can be rewritten as

$$d\mathbf{c} + i(\mathbf{a} - \mathbf{b}) \wedge \mathbf{c} = \mathbf{0} \tag{20}$$

where $\mathbf{a} = a_i\, dx^i$, $\mathbf{b} = b_i\, dx^i$, and $\mathbf{c} = c_i\, dx^i$. Next we write down a solution to equations (19) and (20) that can be shown later to have no magnetic charge and a nonvanishing electric charge.

We introduce the complex coordinate $\zeta = \mathbf{P}_r(r, \theta, \varphi)$ that comes from the stereographic projection $\mathbf{P}_r$ of the spherical polar coordinate (r, θ, φ) on the two-sphere of radius r minus the north pole, $S_r^2 - \{N\}$, to the complex plane minus infinity, $\mathbf{CP}^1\backslash\{\infty\}$. The projected 1-forms $\mathbf{a}$ and $\mathbf{b}$ on $\mathbf{CP}^1\backslash\{\infty\}$ will also be denoted by $\mathbf{a}$ and $\mathbf{b}$. Assume that $\mathbf{a}$ and $\mathbf{b}$ on $\mathbf{CP}^1\backslash\{\infty\}$ differ by the nonexact real-valued 1-form χ; that is, $\mathbf{b} = \mathbf{a} + \chi$. Now define

$$\mathbf{c}(\zeta) = i\sqrt{2}\left[\exp i \int_{\mathbf{P}_\gamma} (\mathbf{b} - \mathbf{a})\right] d\zeta \tag{21}$$

where the contour integration is along the stereographic projection of the great circle γ from the south pole of S_r^2 to the spherical polar coordinate given by $(r, \theta, \varphi) = \mathbf{P}_r^{-1}(\zeta)$. Assume that both the 1-forms $\mathbf{a}$ and $\mathbf{b}$ vanish at the south pole for all values of r. It is easy to verify using the fundamental theorem of calculus that (21) satisfies equation (20) and that $\mathbf{c}^* \wedge \mathbf{c} = 2i\, d\chi \equiv -2d\bar{\zeta} \wedge d\zeta$.

Turning to the Higgs fields, recall that we require that A be $su(2)$-valued everywhere, and that the gauge group for B is broken to $U(1)$ asymptotically ($D^B\Phi_B \to 0$ as $r \to \infty$). Since the Higgs field Φ_A takes values in the Lie algebra of $SU(2)$, then $\|\gamma_A\|^2 = 1 + \alpha_A\beta_A$. Take $\alpha_A = \beta_A = 0$, so that $\|\gamma_A\|^2 = 1$. For Φ_B, condition 4 states that the Higgs fields $\Phi_B = aI_E$, asymptotically. Substitute (16) into the asymptotic equation $D^B\Phi_B = 0$ to find that on the diagonal

$$d(\alpha_B + \beta_B) = 0, \qquad d\alpha_B = i(\mathbf{c}\gamma_B^* - \mathbf{c}^*\gamma_B) \tag{22}$$

and off the diagonal,

$$d\gamma_B = i((\beta_B - \alpha_B)\mathbf{c} + (\mathbf{a} - \mathbf{b})\gamma_B), \qquad \text{complex conj.} \tag{23}$$

For $|\zeta| >> 1$, let

$$\alpha_B(\zeta, \bar{\zeta}) = a + \frac{1}{(1 + |\zeta|^2)^2} + O((\bar{\zeta}\zeta)^{-3})$$

$$\beta_B(\zeta, \bar{\zeta}) = a + \frac{1}{(1 + |\zeta|^2)^2} + O((\bar{\zeta}\zeta)^{-3})$$

$\alpha_B = \beta_B = a$ asymptotically is consistent with the requirement that $\Phi_B = aI_E$, and satisfies the asymptotic equations (22) and (23). The remaining asymptotic equations for γ_B become

$$c_i\gamma_B^* - c_i^*\gamma_B = 0$$

$$\partial_i\gamma_B + i\gamma_B(b_i - a_i) = 0$$

$\gamma_B = 0$ is a solution to these equations and is also consistent with the requirement that $\Phi_B = aI_E$.

Now we compute the electric charge. The broken gauge far-fields are given by

$$F_A \equiv \langle H^A\Phi_A\rangle = -iF\,\mathrm{tr}(\Phi_A) = 0 \tag{24}$$

$$F_B \equiv \langle K^B\Phi_B\rangle = -iF\,\mathrm{tr}(\Phi_B) = F(\alpha_B + \beta_B)/2$$

The magnetic charge of the solution, given by $\lim_{r\to\infty} a \int F_A/2\pi$, vanishes because Φ_A is traceless. The solitonic electric charge is given by

$$2\pi q = a \lim_{r\to\infty} \int_{S_r^2} F_B = -2ia \int_{\mathbf{CP}^1} \frac{d\bar{\zeta} \wedge d\zeta}{(1 + |\zeta|^2)^2} = -4\pi a$$

where $\mathbf{a}$ is closed, but $\mathbf{b}$ is not closed. The sphere S_r^2 is centered around the monopole at the origin and is assumed to lie completely in regions where the gauge field has been broken. Notice that the electric charge within S_r^2 depends only on the gauge potential, and is time-independent, gauge-invariant, and unchanged under any continuous deformation of the enclosing surface. This demonstrates the existence of an electric monopole within the classical field theory.

Which particles, if any, might these solitons be? Note that $U(2)$ is double covered by $SU(2) \times U(1)$. Also, the classical mass and particle spectrum of the projectively-flat electric monopole compare favorably with that of the intermediate vector bosons. The mass of the electric monopole is $M = aq$, the same as the mass of the $W^{\pm}$. Moreover, there are no quantum corrections to the classical mass, because of the general relationship between supersymmetry and the Bogomol'nyi structure (Witten and Olive, 1978; Hlousek and Spector, 1993). The Z_0 presumably corresponds to the case where $H^A = K^B = FI_E$, but $D^A\Phi_A \neq 0$ and $D^B\Phi_B \neq 0$, that is, there is no symmetry breaking. The gauge far-fields in that case are non-Abelian and would therefore pass unnoticed through the detectors. Although uncharged, the soliton's energy is topologically fixed by

$$2\pi\mathscr{E} = \int_{S^2} \operatorname{tr}(\Phi_A H^A) + \int_{S^2} \operatorname{tr}(\Phi_B K^B)$$

$$= -\int_{S^2} F\langle\Phi_A\rangle - \int_{S^2} F\langle\Phi_B\rangle$$

So it, too, is stable under perturbations. We propose therefore that the solitons in the tensor product topological field theory defined in Section 2 form a provisional model for the $W^{\pm}$ and Z_0 intermediate vector bosons. Recall that many years ago Montonen and Olive (1977) found quantum evidence to suggest that intermediate vector bosons should appear as Bogomol'nyi solitons, dual in some sense to the BPS magnetic monopole.

5. CONCLUSION

The tensor product energy functional (2) has a Bogomol'nyi structure and solitonic particle solutions. Since the Bogomol'nyi equations do not arise from a perfect square, there is more flexibility in achieving the Bogomol'nyi energy. A class of solitons has been investigated in some detail by restricting to those solutions of the Bogomol'nyi equations where the gauge potentials are equal, $A = B$. When both Bogomol'nyi equations are satisfied stable, particle-like solitons are dyonic. By relaxing the Bogomol'nyi structure, stable, particle-like solutions can be found that carry only an electric charge. Similarities exist between the heavy intermediate vector bosons in the standard model and the solitons in this model.

As with the BPS magnetic monopole, the moduli space of electric monopoles is the configuration space of the physical theory. At present the moduli space is too large to be useful. This is easy to see: aside from being differentiable on $\mathbf{R}^3$ and some asymptotic boundary conditions, we demand nothing else from the Higgs fields in Section 4. Further analytic or geometric structure is clearly required. We note that the projective-flatness given to us by the Bogomol'nyi equations is, on certain holomorphic vector bundles, equivalent to algebraic stability. Algebraic stability is frequently employed to construct well-behaved moduli spaces (Kobayashi, 1987).

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